ARC Centre of Excellence for Antimatter-Matter Studies

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Fluid Modelling of Plasmas and Swarms: The Big Picture

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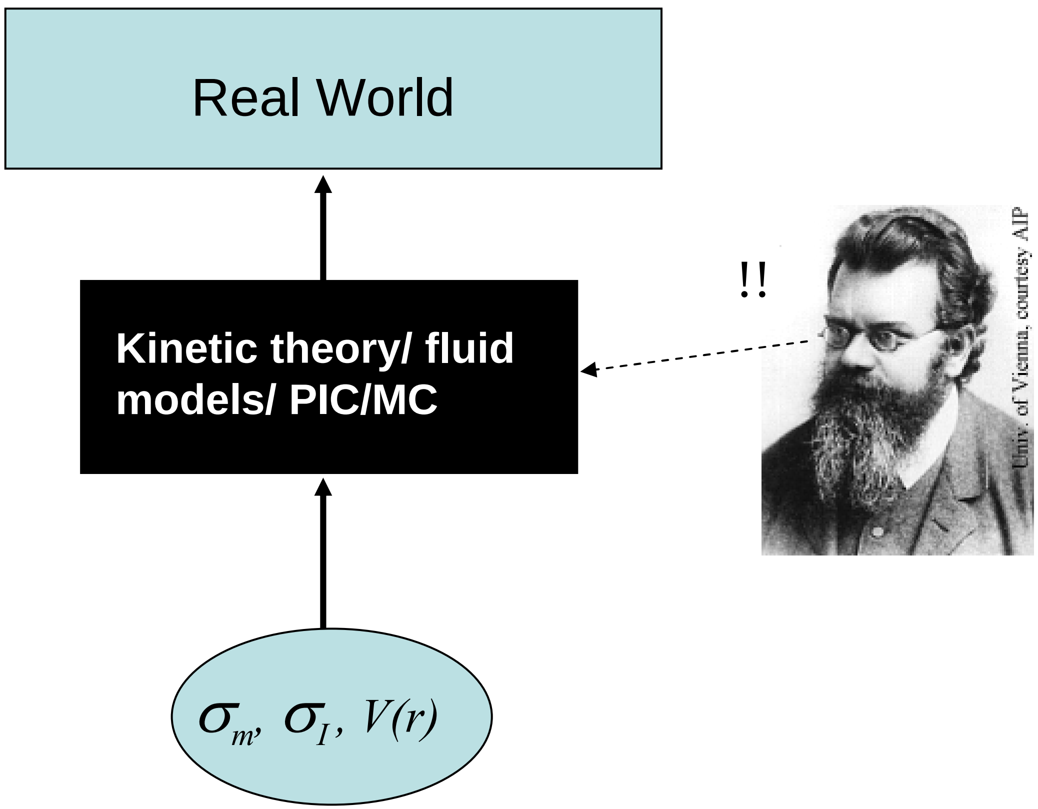
Microscopic → Macroscopic



Kinetic Theory

- Since systems are often far from equilibrium $f \neq$ Maxwellian
- Mean free path analysis too crude to be of much use
- Must solve Boltzmann's equation for the phase space distribution function $f(\mathbf{r},\mathbf{c},t)$
- Calculate measurable quantities as velocity "moments", *e.g.,* average velocity

$$< \mathbf{c} > = \int \mathbf{c}\, f(\mathbf{r},\mathbf{c},t)\, d^3\mathbf{c} \,/ \int \mathbf{c}\, f(\mathbf{r},\mathbf{c},t)\, d^3\mathbf{c}$$

- Boltzmann's equation plays same role in statistical mechanics as Schrödinger's equation in quantum mechanics, e.g.,

$\psi(\mathbf{r},t)$ + calculation of expectation values <…>

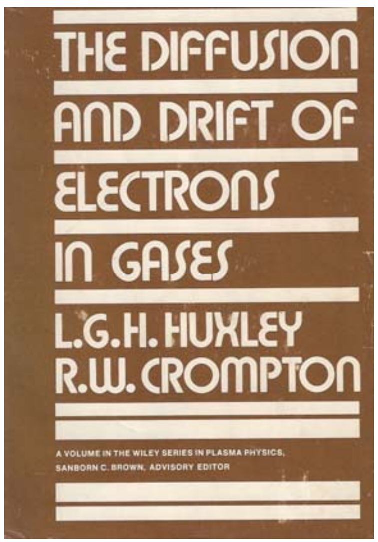
Kinetic Theory of $e^{\pm}$, $\mu^{\pm}$, $A^{\pm}$ in gases

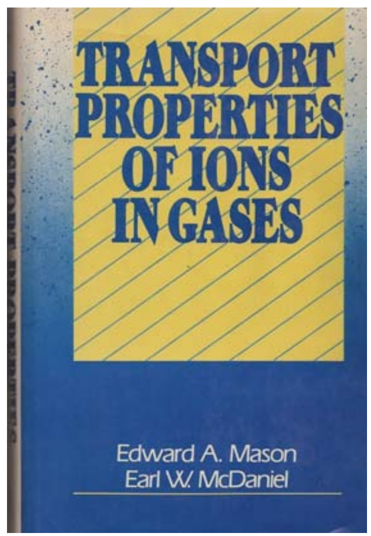
Modern era

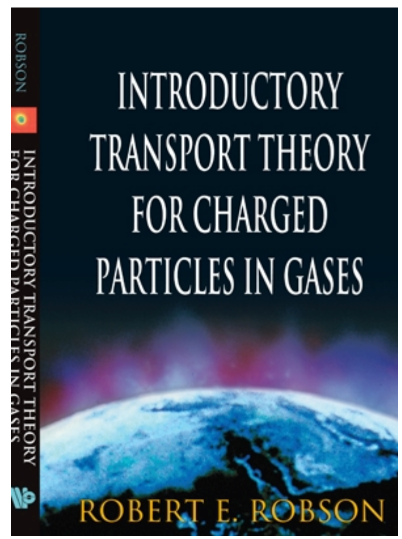
t > 1975 (ions), *t > 1979* (electrons, positrons, muons)

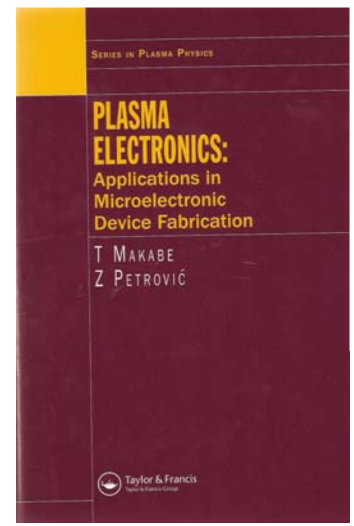
History

- *Boltzmann 1872* (classical elastic collisions) → Fokker-Planck equation
- *Wang-Chang, Uhlenbeck, de Boer 1951* (semi-classical inelastic collisions)
- *Waldmann (1958) - Snider (1961)* (quantum kinetic equation)









1974 1988 2006

Fluid Modelling

• “Short cut" method → <... > directly **without** solving Boltzmann’s equation - projection of phase space $(\mathbf{r},\mathbf{c},t)$ onto configuration space $(\mathbf{r},t)$

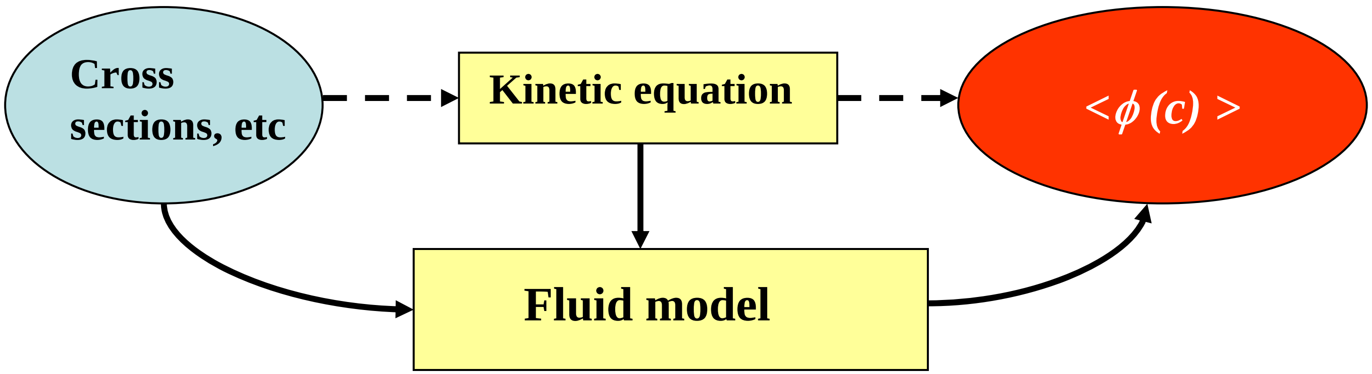
• Motivation - **computationally economical**

• Fluid equations ≡ adaptation of $\boldsymbol{F} = m\,\boldsymbol{a}$ to fluids + closure *assumptions* to make equations useful

• Numerous *ad hoc*, unphysical assumptions → proliferation of fluid models

Towards a Physically-based Fluid Model

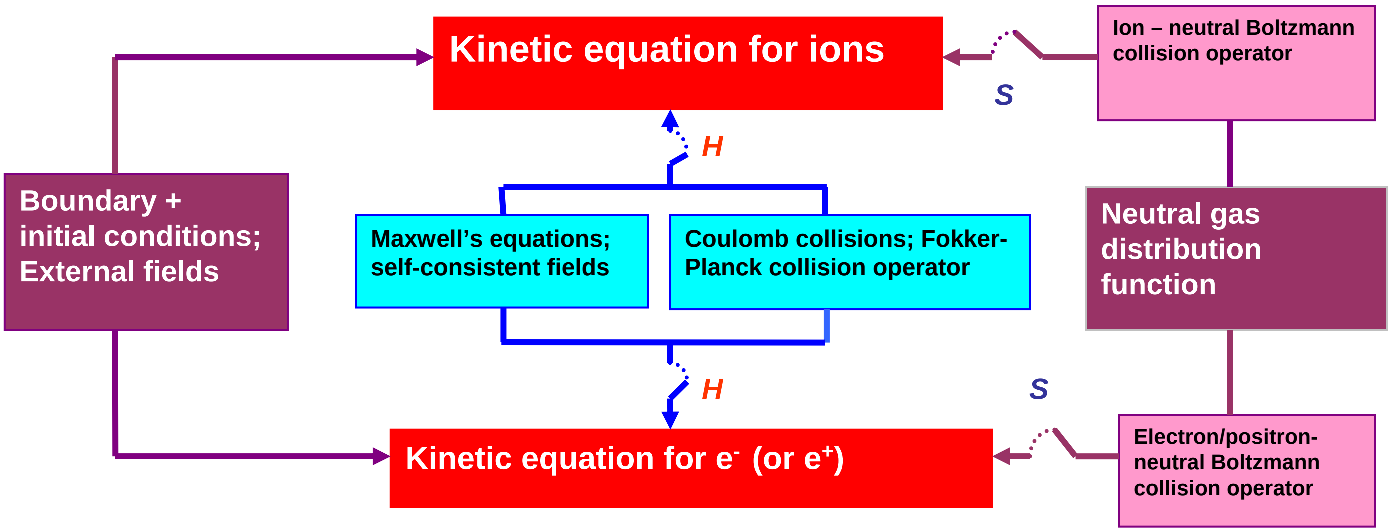
Aim: To develop physically-based equations valid for *all* types of charged species ($e^{\pm}$, $A^{\pm}$, …)



- Form balance equations $\int d\mathbf{c}\ \phi_i(\mathbf{c}) \times$ *Kinetic equation*
- Systematic approximation required to → useful equations
- Benchmarking required to establish accuracy
- " One size fits all" possible? After all, there is only one $F = ma$

Robson, White and Petrović, Rev. Mod. Phys. (2005)

Swarm and plasma modelling



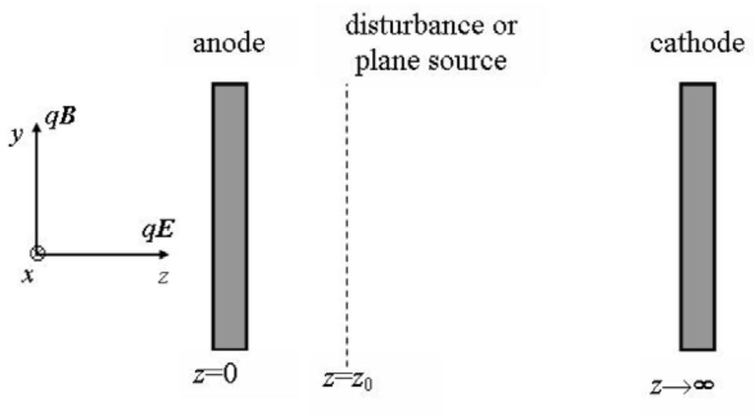
• *Swarm* (= "test particle") limit - "switches" ***H*** are open, while the "switches" ***S*** are closed. Collective effects are usually absent in this limit $(\lambda_D > L)$

• *Hot, fully ionized plasmas* - "switches" ***H*** are closed while the "switches" ***S*** are open. Coulomb interaction and collective effects (waves, instabilities) dominate $(\lambda_D << L)$

• *Low temperature, collision-dominated plasmas* are intermediate between these two limits, and generally *all* factors must be taken into consideration, i.e., all "switches" ***S*** +***H*** are closed

Benchmark Model

Focus on *relaxation phenomena* in plane-parallel geometry
(G. Petrov et al 1996; B. Li *et al*, J Phys B 2000; Phys Rev E 2006)

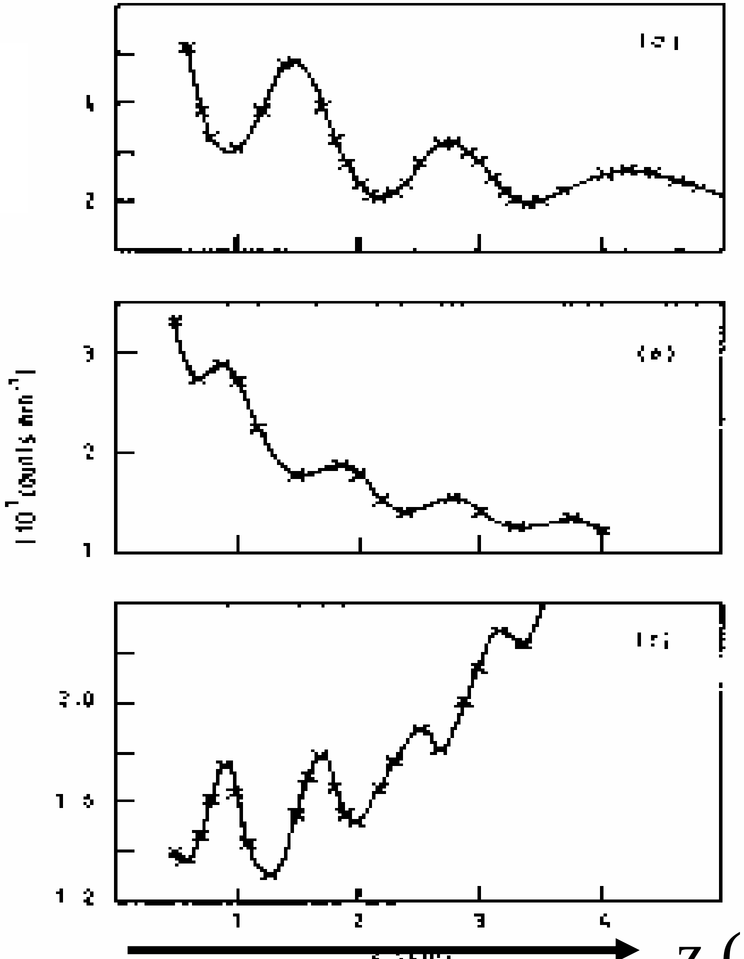


- Electrons emitted at a steady rate from source into infinite gas
- Swarm limit, external fields, no ionisation or attachment
- Diffusion equation → unphysical results (both ions and electrons)
- Only a carefully formulated fluid model → physically meaningful results
- Exact analytical solution of Boltzmann equation known for ν_m =constant
- Solution of Boltzmann equation for real gases - accuracy ~ 2%

Fletcher J. Phys. D(1985) (photon flux technique)

Intensity

(10^3 counts/min)



Ne

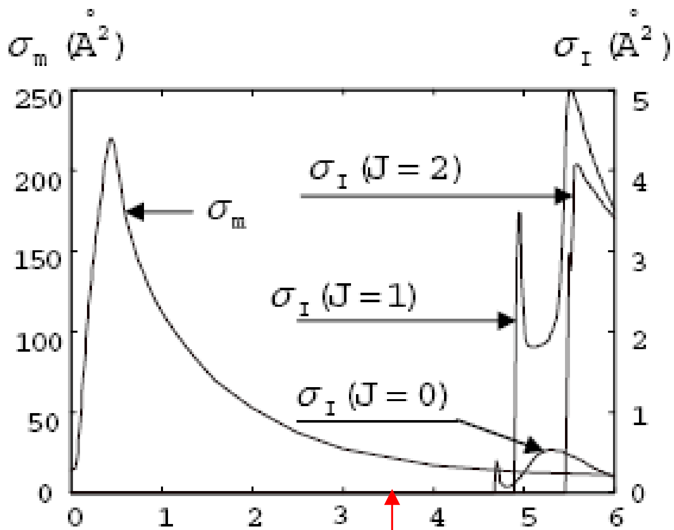
Ar

He

z (cm)

Oscillations occur only in a 'window' of voltages and gas pressures

(e^-, Hg) Franck-Hertz oscillations

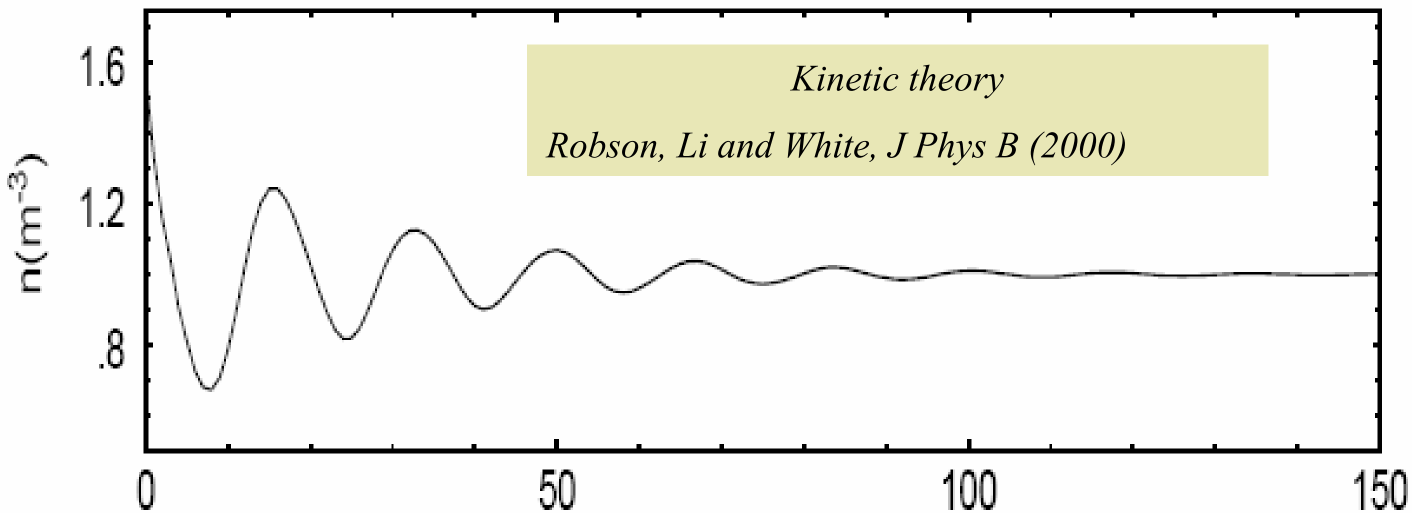


Cross sections

Hanne, Amer. J. Phys. (1988)

England and Elford, Aust. J. Phys. (1991)

Ps formation threshold



Normalised distance from the cathode →

Fluid Equations

$$\frac{\partial \Gamma}{\partial z} = 0$$

$$\frac{2}{3}\frac{\partial (n\varepsilon)}{\partial z} = neE - nm\nu_m(\varepsilon)\mathrm{v}$$

$$-\frac{1}{\nu_e}\left[\mathrm{v}\frac{\partial \varepsilon}{\partial z} + \frac{2\varepsilon}{3}\frac{\partial \mathrm{v}}{\partial z} + \frac{1}{n}\frac{\partial J_q}{\partial z}\right] = \varepsilon - \frac{1}{2}M\mathrm{v}^2 + \Omega(\varepsilon)$$

Inelastic collision term $\Omega = \sum_I \epsilon_I \left(\overrightarrow{\nu_I} - \overleftarrow{\nu_I}\right) / \nu_e(\varepsilon)$

To *close* the equations, need to express heat flux J_q in terms of n, v, ε

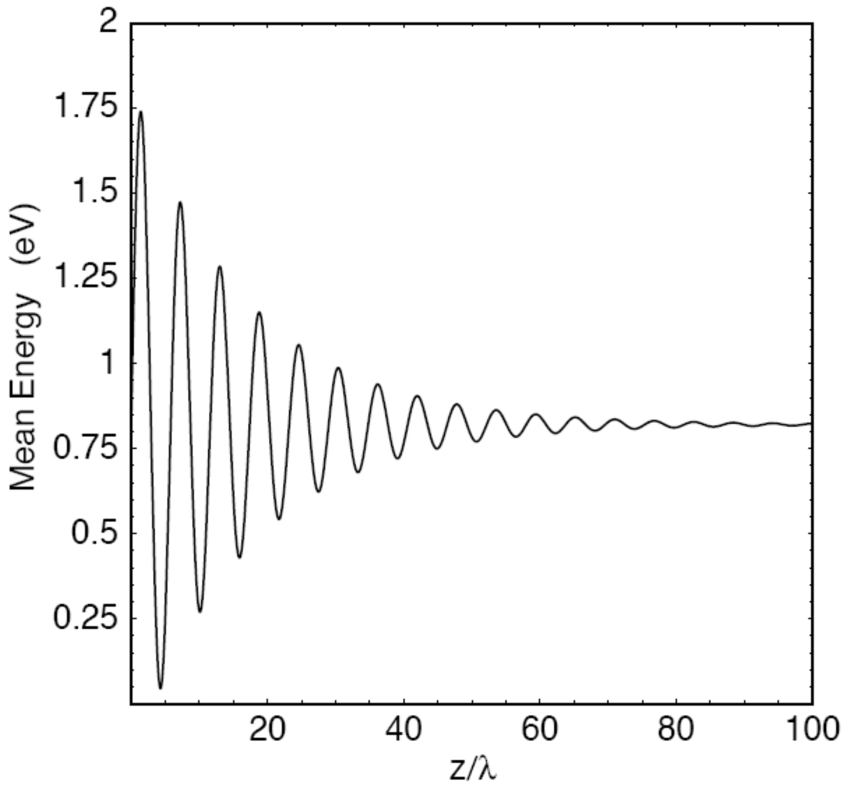
Ansatz $J_q = -\frac{2}{3m}\frac{\partial}{\partial z}\left[\frac{n\xi(\varepsilon)}{\nu_m(\varepsilon)}\right] + \frac{(5-2p)}{3}\frac{na\varepsilon}{\nu_m(\varepsilon)} - \frac{5}{3}\Gamma\varepsilon$ $\left(p = \frac{d\ln \nu_m}{d\ln \epsilon}\right)$

Expression *exact* for $\nu_m(\varepsilon) \sim \varepsilon$ and then $\xi(\varepsilon) = \varepsilon^2$

- Fourier ansatz $J_q = -\lambda\, \partial\varepsilon/\partial z$
- $J_q = 0$

Surendra and Dalvie (1993)

Mean energy in the "window"



Mean electron energy at ***E/N* = 6.5 Td** as a function of normalized distance downstream from the source. These Franck-Hertz oscillations are characteristic of all physical properties in the "window" region of *E/N* - mean velocity, number density, heat flux, although there are phase differences

References

Kinetic modelling

'Advances in low temperature r.f. plasmas' , Ed. T. Makabe (2002)

Robson, White and Morrison, J Phys B 36, 4127 (2003)

Porokhova et al, Phys Rev E 71, 066406 and 066407 (2005)

Pinhao et al, PSST 13, 719 (2005)

Petrović et al, PSST 16, S1-12 (2007)

Fluid Modelling

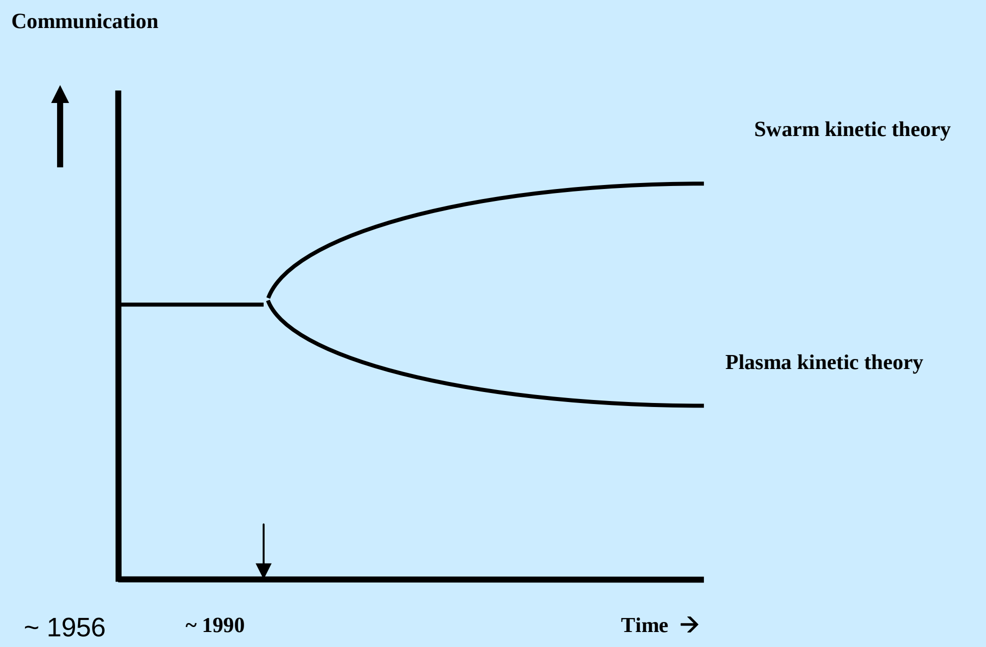
Hagelaar and Pitchford, PSST 14, 722 (2005)

Robson et al, Rev. Mod. Phys. 77, 1303 (2005)

Viehland and Goeringer, J. Phys. B 38, 3987 (2005)

Viehland, GEC Invited Talk (2006)

Swarm vs Plasma Literature



Why?

Asymptotic Regime

Linearize equations far downstream from source: $n(z) = n_\infty + n_1\, e^{Kz}$ etc.

Three equations → *cubic* secular equation, dimensionless "wave number" $K = \frac{3}{2}\frac{eE}{\varepsilon_\infty}\kappa$

$$-\frac{3}{2}\alpha(\bar{p} - p - 1)\kappa^3 + \left(\frac{3}{2}\alpha\bar{p} - \frac{5}{2}p + p^2\right)\kappa^2 + \left(\gamma + 2p - \frac{7}{2}\right)\kappa - (\gamma + 2p) = 0$$

$$\bar{p} = \frac{\varepsilon_\infty\, \xi'(\varepsilon_\infty)}{\xi(\varepsilon_\infty)}$$

$$\alpha = \frac{\xi(\varepsilon_\infty)}{\varepsilon_\infty^2}$$

Key parameter $\gamma = \dfrac{(1+\Omega')\varepsilon_\infty}{\varepsilon_\infty + \Omega}$

Benchmark model